

TEMA 3 b) SISTEMAS DE EDO(1) L

$$\frac{dx_1(t)}{dt} = a_{11}x_1(t) + a_{12}x_2(t) + b_1(t) \quad x_1(0) = c_1$$

$$\frac{dx_2(t)}{dt} = a_{21}x_1(t) + a_{22}x_2(t) + b_2(t) \quad x_2(0) = c_2$$

$$\bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \frac{d}{dt} \bar{x}(t) = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \bar{x}(0) = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad \bar{b}(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}$$

$$\frac{d}{dt} \bar{x}(t) = A \times \bar{x}(t) + \bar{b}(t)$$

$$\frac{dx_1(t)}{dt} = 2x_1(t) + 3x_2(t)$$

$$\frac{dx_2(t)}{dt} = 4x_1(t) + 5x_2(t)$$

5(2) EDO(1) LCC H

$$4x_1(t) = \frac{dx_2(t)}{dt} - 5x_2(t)$$

$$\Rightarrow x_1(t) = \frac{1}{4} \left(\frac{dx_2(t)}{dt} - 5x_2(t) \right)$$

$$\frac{dx_1(t)}{dt} = \frac{1}{4} \left(\frac{d^2x_2(t)}{dt^2} - 5 \frac{dx_2(t)}{dt} \right)$$

$$\frac{dx_1(t)}{dt} = 2x_1(t) + 3x_2(t)$$

$$\frac{1}{4} \left(\frac{d^2x_2(t)}{dt^2} - 5 \frac{dx_2(t)}{dt} \right) = \frac{2}{4} \left(\frac{dx_2(t)}{dt} - 5x_2(t) \right) + 3x_2(t)$$

$$\frac{d^2x_2(t)}{dt^2} - 5 \frac{dx_2(t)}{dt} = 2 \frac{dx_2(t)}{dt} - 10x_2(t) + 12x_2(t)$$

$$\frac{d^2x_2(t)}{dt^2} + (-5-2) \frac{dx_2(t)}{dt} + (10-12)x_2(t) = 0$$

$$\frac{d^2x_2(t)}{dt^2} - 7 \frac{dx_2(t)}{dt} - 2x_2(t) = 0$$

$$m^2 - 7m - 2 = 0$$

$$m = \frac{7 \pm \sqrt{49+8}}{2} \Rightarrow \frac{7 \pm \sqrt{57}}{2}$$

$$(m - 14.54)(m + 0.54) = 0$$

$$x_2(t) = c_1 e^{14.54t} + c_2 e^{-0.54t}$$

$$x_1(t) = \frac{1}{4} \left(\frac{dx_2(t)}{dt} - 5x_2(t) \right)$$

$$\Rightarrow \frac{dx_2}{dt} = 14.54 c_1 e^{14.54t} - 0.54 c_2 e^{-0.54t}$$

$$x_1(t) = \frac{1}{4} \left(14.54 c_1 e^{14.54t} - 0.54 c_2 e^{-0.54t} - 5 c_1 e^{14.54t} - 5 c_2 e^{-0.54t} \right)$$

$$x_1(t) = \left(\frac{14.54}{4} c_1 - 5 c_1 \right) e^{14.54t} + \left(-0.54 c_2 - 5 c_2 \right) e^{-0.54t}$$

$$\frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} - 6y = 0$$

$$y(x) = y_1(x)$$

$$\frac{dy}{dx} = \frac{dy_1(x)}{dx} = y_2(x)$$

$$\frac{d^2 y}{dx^2} = \frac{dy_2(x)}{dx} = y_3(x)$$

$$\frac{d^3 y}{dx^3} = \frac{dy_3(x)}{dx} = 6y_1(x) - 4y_2(x) + 2y_3(x)$$

$$\frac{d}{dx} \begin{bmatrix} y_1(x) \\ y_2(x) \\ y_3(x) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -4 & 2 \end{bmatrix} \times \begin{bmatrix} y_1(x) \\ y_2(x) \\ y_3(x) \end{bmatrix}$$

$$\frac{d}{dx} \bar{y} = A_x \bar{y}$$

$$\left. \begin{array}{l} \frac{dx_1}{dt} - 6 \frac{dx_2}{dt} = 4x_1 \\ 3 \frac{dx_1}{dt} + 12 \frac{dx_2}{dt} = 6x_2 \end{array} \right\} \begin{array}{l} 2 \frac{dx_1}{dt} - 12 \frac{dx_2}{dt} = 8x_1 \\ 3 \frac{dx_1}{dt} + 12 \frac{dx_2}{dt} = 6x_2 \end{array}$$

$$5 \frac{dx_1}{dt} = 8x_1 + 6x_2$$

$$\rightarrow \frac{dx_1}{dt} = \frac{8}{5}x_1 + \frac{6}{5}x_2$$

$$\begin{array}{r} -3 \frac{dx_1}{dt} + 12 \frac{dx_2}{dt} = -12x_1 \\ 3 \frac{dx_1}{dt} + 12 \frac{dx_2}{dt} = 6x_2 \end{array}$$

$$30 \frac{dx_2}{dt} = -12x_1 + 6x_2$$

$$\rightarrow \frac{dx_2}{dt} = -\frac{12}{30}x_1 + \frac{6}{30}x_2$$

$$\rightarrow \begin{array}{l} \frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 \\ \frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 \end{array}$$

$$\frac{d}{dt} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & 9 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 5e^{2t} + 6t^2 + 4\cos(3t) \\ 6te^{3t} + 4t + 2 - 2\sin(x) \\ 12e^{2t} + 4t^2 + 5\cos(x) \end{bmatrix}$$